Using transport simulations in comparison to experimental data in order to constrain the nuclear equation of state and nucleon effective mass splitting

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The nuclear equation of state (EoS) near saturation density has been constrained for some time, however the behavior at significantly sub- and super-saturation densities is still poorly constrained. Numerous simulations suggest that the ratio of the kinetic energy spectra of neutrons and protons would be sensitive to both the symmetry energy as well as the nucleon effective mass splitting, should any exist. This ratio is defined as $SR\left(\frac{n}{p}\right)=\frac{Y(n)}{Y(p)}$ where $Y(X)$ is the kinetic energy spectrum of particle X. This yield can also be represented by the double differential $\frac{dM}{d\Omega d(E/A)}$. In the case of a stronger symmetry energy, neutrons would be accelerated away from a neutron rich source while protons would be attracted towards the same source. In a similar fashion, if $m_n^* < m_p^*$ the neutrons would feel a stronger force than the protons and be accelerated to larger energies. By making comparisons it should be possible to constrain these quantities.

One difficulty with the n/p ratio is the difficulty of accurately measuring both the neutron detection efficiency as well as the effects due to Coulomb acceleration from the source. There are three primary options that can be used in order to minimize these effects. The first is the independent particle ratio, $IPR(X)=\frac{Y_{\delta_2}(X)}{Y_{\delta_1}(X)}$ where $\delta=\frac{N-Z}{A}$ of the initial system. In this case δ_1 is defined as a system, typically symmetric or as close as possible while still being stable enough to use as a target, and δ_2 a more neutron rich system. This ratio can minimize the effects of detection efficiency as well as some Coulomb effects. This ratio also has a potential downfall because it does not compare particles of different isospin. The next option is to use the double ratio, defined as $DR\left(\frac{n}{p}\right)=\frac{IPR(n)}{IPR(p)}=\frac{SR_{\delta_2}(n/p)}{SR_{\delta_1}(n/p)}$. This takes advantage of the effects of the IPR while still comparing particles of different isospin.

The final option is to use different particles to measure each of these ratios. In particular the particles that have been suggested are mirror nuclei including the A =3, 7, 11, 15 and 19 pairs. There has been some debate over whether or not these mirror nuclei should be sensitive to the symmetry energy and nucleon effective mass splitting.

Recent experimental evidence implies an answer to this debate with the following derivation. For this treatment, a central collision resulting in a single expanding source is used. This collision results in a high density neutron rich environment. As the system expands the density will decrease to a point where clusters can begin to coalesce and bind together. This density was experimentally measured to exist in a region of densities $\rho < \rho_0/10$ [1]. The symmetry energy and the effects of the effective mass splitting at these densities should be negligible in comparison to the effects that existed in the high density regions.

In the Awes model of clustering [2], the spectra at the point of emission of a cluster with Z protons and N neutrons can be directly compared to the spectra at emission of protons ($Y(p)$) and neutrons ($Y(n)$) using the following form,

$$Y(Z,A) = Y(p)^Z Y(n)^N f(Z,A)$$

where $f(Z,A)$ is an energy dependent scaling factor. This scaling factor can be thought of as a coalescence volume in momentum space. In principle it is possible for the scaling factor to be different from one collision system to another, however for the systems of interest in this work the scaling factor will be assumed to be the same due to the relatively small differences between the systems. This indicates that the IPR for deuterons, the simplest cluster, would be

$$IPR(d) = \frac{Y_{\delta_2}(d)}{Y_{\delta_1}(d)} = IPR(n)IPR(p)$$

Since one would not expect the symmetry energy to affect symmetric particles like deuterons and alphas, it would be predicted that $IPR(d) = IPR(\alpha) = 1$ from which the result that $IPR(n) = 1/IPR(p)$ would naturally follow. Using this result and investigating the $A = 3$ clusters, the predicted value for those particle ratios provides

$$IPR(t) = IPR(p)IPR(n)^2 = IPR(n)$$
$$IPR(h) = IPR(p)^2 IPR(n) = IPR(p)$$

This method can continue upwards in cluster size indicating that for every isotope with $N - Z = 1(-1)$ that isotope's independent particle ratio should be equivalent to that of neutrons(protons). As a consequence, it also suggests that $DR(n/p) = IPR(n)^2$. There exists experimental evidence supporting these assumptions for clusters up through alpha particles in Ref [3].

This result has two significant impacts. This result suggests that the double ratio of any mirror nuclei that differ by a single neutron should be equally as sensitive to the symmetry energy and nucleon effective mass splitting as the n/p ratio. If this holds true, then it indicates that measuring the kinetic energy of neutrons, a notoriously difficult task, is not necessary to constrain either quantity so long as a sufficient measurement of charged particles is obtained. To date this treatment has been applied to systems that specifically measured light clusters (up through$A \leq 4$) but were not applied to larger clusters.

In order to test the validity of this theory for larger clusters the experimental data obtained by Kohley *et al.* and described in Ref [4] will be used. While research on this project is ongoing several comparisons can be made with initial existing data. A test of the validity of this cluster model can be immediately made by investigating the IPR of different particles. In order to attempt to constrain the symmetry energy, comparisons can be made between the spectra and ratios from ^{64}Zn+^{64}Zn and ^{70}Zn+^{70}Zn both at 35 MeV/A to simulations using the AMD [5] and CoMD [6,7] transport codes. In both simulations, the results are passed through an experimental filter in order to directly compare the number of counts per event between the simulated and experimental results. In all cases, only particles emitted in the range of $70 \leq \theta_{CM} \leq 110$ are considered so that effects from secondary breakup and decay are minimized. In addition, the events considered are those deemed most violent, which at this time are assumed to be the most central collisions.

The first comparisons involve the IPR of different particles in the system to test the validity of the coalescence. The IPR for several pairs of mirror nuclei as well as symmetric particles are provided in Fig. 1. These ratios are generated by taking the raw counts per event for each system. This is a very raw measurement with future considerations needed such as compensation for detector losses, multiple hit probabilities and changes in geometric acceptances between the two systems; however even with this raw measurement some interesting observations can be made. In the "proton-like" plot (left panel), the three lightest clusters, ^{1}H, ^{3}He and ^{7}Be, show very good agreement. The three larger clusters have a different value but display the same flat behavior. The "deuteron-like", or symmetric particles (middle panel), all agree quite well up through 40 MeV, at which point the alpha particles deviate significantly for an as yet not understood reason. The "deuteron-like" ratio is approximately 1.1, however efficiency corrections have not yet been applied. Finally, the "neutron-like" particles (right panel) show one of the largest ranges of results, however, with the exception of the tritons all behave quite similarly. This collection of results which is admittedly still very raw provides some evidence to support the coalescence predictions and encourages further pursuit.

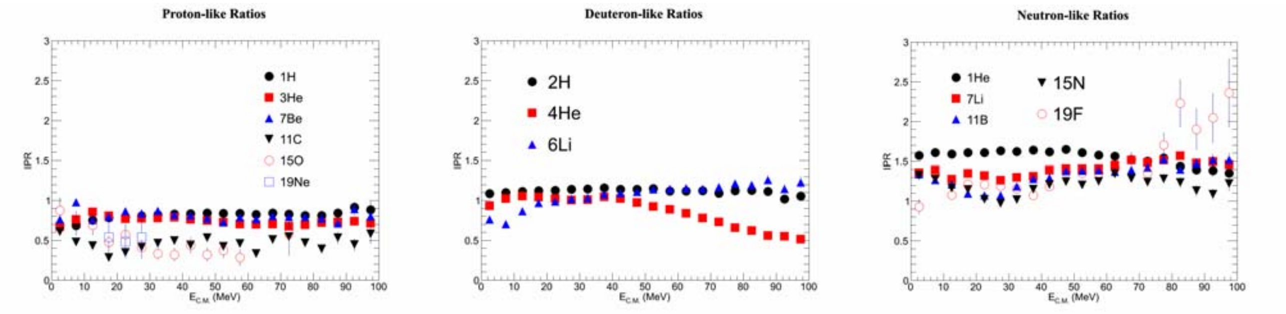

FIG. 1. The "proton-like", "deuteron-like" and "neutron-like" particle ratios are shown in order from left to right.

The ultimate goal of this analysis is to attempt to constrain the symmetry energy and/or the nucleon effective mass splitting by comparing the results to transport calculations. Since neutron energy spectra was not measured in this experiment, the most basic comparison would be made using the t/h double ratio. The spectra of tritons (left) and helions (right) from the ^{64}Zn+^{64}Zn collision that are used in constructing both the double ratio and the independent particle ratio are provided in Fig. 2. In both cases they are compared to a CoMD simulation using a roughly linear form for the symmetry energy and an AMD simulation using the GognyAS parameterization. For both simulations, an experimental filter has been applied in an attempt to recreate experimental conditions and is compared to the raw counts per event from the experimental results. It is immediately obvious that the simulated spectra provide a distinctly different shape that the experimental results.

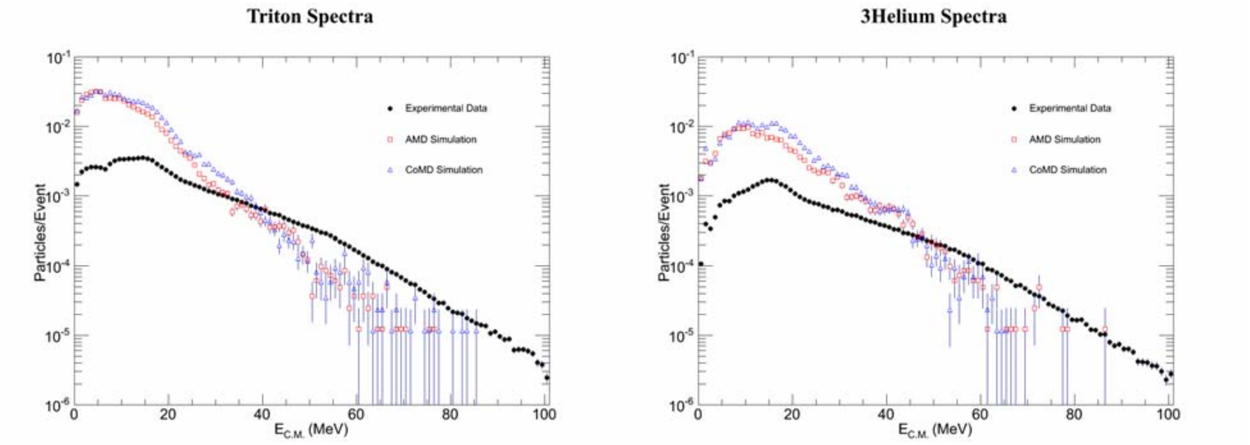

FIG. 2. The spectra of measured tritons (left) and helions (right) in comparison to simulated AMD and CoMD predictions.

This is a potentially dangerous result as it is tempting to use the double ratio to constrain the symmetry energy since it is possible to provide reasonable comparisons between theoretical and experimental results despite being built from spectra that are unreasonable in comparison. Any attempt to extract constraints on the symmetry energy from the double ratio or independent particle ratios need to be tempered until the discrepancy between the simulated and measured kinetic energy spectra can be understood.

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